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Minimizing Blade Dynamic Response in a Bladed Disk Through Design Optimization

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A study is presented on minimizing the maximum dynamic response in a mistuned bladed disk through design optimization. A well-studied spring–lumped-mass system was used to model a bladed disk, and the problem was formulated as a constrained, nonlinear optimization process. Intentional mistuning is introduced by varying the blade mass within a given range. An intentional mistuning pattern described in a polynomial form was then solved iteratively to search for the optimized mistuning pattern that produces the smallest maximum blade response amplitude over a given range of excitation frequencies. It was found that the dynamic amplification factor of the maximum responding blade can be reduced to a range between 20 and 40% less than the tuned system for several combinations of engine excitation orders and coupling ratio. The comparison of results shows that this reduction is more effective than the harmonic or linear mistuning patterns proposed in the literature. The effectiveness of the optimized mistuning patterns was examined through Monte Carlo simulations. The optimized mistuning patterns were found to reduce the maximum blade response for all engine excitation orders in the presence of random mistuning. Hence, it may be possible to reduce significantly the maximum blade response levels in bladed disks by implementing an optimized intentional mistuning pattern.

Nomenclature

A	=	amplitude of harmonic mistuning
a_n	=	nth coefficient of a polynomial
c_b	=	viscous damping of the ith blade without damping mistuning
c_c	=	viscous damping due to blade to disk coupling
c_i	=	viscous damping of the ith blade
d_i	=	amplification factor for the ith blade
E	=	engine excitation order
F_0	=	force amplitude of engine excitation
f_i	=	excitation force applied to the ith blade
h	=	harmonic order
j	=	$\sqrt{-1}$
k_c	=	coupling stiffness of the ith blade
k_i^b	=	stiffness of the ith blade
l	=	order of polynomial
m_b	=	mass of the ith blade without mistuning
m_i	=	total mass of the ith blade
N	=	number of blades
R	=	coupling ratio, $R^2 = \omega_c^2/\omega_b^2$
t	=	time
X_i	=	response amplitude of the ith blade in mistuned system
X_i^0	=	response amplitude of the ith blade in tuned system
x_i	=	displacement of the ith blade
x_i^0	=	displacement of the ith blade in tuned case
δ_i	=	mistuning strength for the ith blade
δ_i^{limit}	=	limit of mass mistuning for the ith blade
θ	=	interblade angle
ξ	=	damping ratio
$\mathbf{\Phi}$	=	N by 1 vector, excitation phase vector
$\phi_{i,k}$	=	ith element of the kth eigenvector
$\{\phi^k\}^T$	=	1 by N vector, transpose of the eigenvector for kth vibration mode
ω	=	fundamental excitation frequency
ω_b	=	blade frequency
ω_c	=	coupling frequency
$\omega_{n,k}$	=	natural frequency of kth vibration mode

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I. Introduction

PERIODIC structures such as turbine engine bladed disks generally have families of closely spaced and repeated vibration modes. This type of eigenstructure is very sensitive to small perturbations in geometry and material properties, which inevitably occur due to manufacturing tolerances or property deterioration of components during service. These small perturbations, defined as mistuning, usually increase the maximum blade forced response amplitude. As a result, mistuning may cause high-vibration stresses and can have detrimental impact on the high cycle fatigue (HCF) life of the component. To date, a large amount of research has been carried out in understanding mistuning and the associated negative impact.[1–17] Two approaches have been used to evaluate the effect of mistuning on blade HCF life.

The first approach is to predict the maximum amplitude of the blade response under any combination of mistuning patterns.[1,3,4,14,15,18] The maximum amplification of the blade response over the tuned system was first reported by Whitehead[1,4,18] as $(1 + \sqrt{N})/2$ (where N is the number of blades) for all engine orders, and Feiner et al.[15] recently confirmed Whitehead's limit. However, designing a bladed disk purely based on the worst-case scenario is very conservative for a large percentage of the blade population.

The second approach stems from considering whether an intentional mistuning could be found and implemented into a nominal design of a bladed disk so that the amplitude magnification due to random mistuning could be reduced. This concept has been investigated in several studies, and some promising results have been achieved through implementing various intentional mistuning patterns. For instance, an arrangement of a third-order cosine pattern[2] and an alternating high and low blade frequency pattern around a disk[5] were found to be able to reduce blade response amplitudes, although intentional mistuning may significantly affect bladed disk flutter behavior.[19,20] A further study indicated that harmonic mistuning patterns were able to reduce the maximum blade response.[21] Using both spring–lumped-mass models and finite element models, Castanier and Pierre[17,22] Brewer et al.,[23] Kenyon and Griffin,[24] and Castanier and Pierre[25] conducted extensive studies on harmonic mistuning and found it to be effective in reducing the blade response sensitivity to additional random mistuning. In a recent study, the best and worst blade arrangements were sought by using a design optimization procedure through building gradient-based response surface approximations.[26] A minimum value of response amplification of 1.25 was achieved for a 33-bladed disk. It is also reported that a reduction of amplitude magnification can be achieved by alternating the properties of two blades using a genetic optimization technique,[27] and an amplitude reduction of 21% below the tuned case was achieved. After evaluating the effectiveness of linear, random and harmonic mistuning patterns from a statistical analysis,[28] Jones and Cross stated that the blade frequency mistuning was most effective at reducing the tuned response for engine orders equal to $N/4$ but not possible for $N/2$ and N engine orders. Also, the linear mistuning pattern was found to be able provide more reduction than a harmonic mistuning pattern. A maximum blade amplitude reduction of 28% below the tuned system was achieved for engine excitation order three.

From the previous research work on intentional mistuning, two fundamental issues can be identified for further examination. The first issue is what intentional mistuning patterns produce the most effective reduction of maximum blade amplitude. Previous work has shown that both a harmonic (or n-periodic) pattern and a linear pattern can produce a blade response level below the tuned system under certain conditions. However, the effectiveness of these mistuning patterns may be limited because these mistuning patterns were intuitively defined. Results from the recent studies on searching for the best intentional mistuning patterns[26,27] have shown significant potential for the reduction of blade response amplitude, but no attention was paid to the effect of the mistuning shape on reducing vibration response amplitude. Furthermore, the solution of the problem may become computationally expensive when all blades are mistuned in practice,[26] or it may not be representative for quantifying the effect of mistuning shape when only two blades were intentionally mistuned.[27] To our knowledge, no attempt has been published in literature to investigate the optimal shape of the intentional mistuning pattern and its effect on the blade vibration amplitude magnification, despite the findings on harmonic and linear mistuning patterns.

The second issue is the effect from the engine excitation order and from the internal coupling between blades and a disk on the amount of amplitude reduction. Although the effect of engine excitation order was addressed using a statistical analysis,[28] the amount of the reduction under other engine orders was not reported. The effect of internal coupling, although considered as an important factor in the literature,[9,10,17] has not been addressed in the context of intentional mistuning as the emphasis was given to the effect of selected mistuning patterns on the reduction of maximum amplitude.

This paper, with a focus on these two issues, aims to reduce the maximum blade response by using a design optimization technique. The problem was formulated as a non-linear constrained optimization procedure. The maximum dynamic magnification factor was chosen as the objective function and the changes in blade mass were selected as the mistuning parameters within given bounds. The mass mistuning pattern was described as a polynomial function of the blade number. Only unknown polynomial coefficients were optimized in order to reduce the number of design variables. The optimized mistuning pattern for each engine order was found by searching for the mistuning pattern that produces the minimum value of the maximum dynamic response. It was found that the peak dynamic amplification factor can be reduced to a range between 20 and 40% less than the tuned system for several combinations of the engine excitation order and the coupling ratio between the blades and the disk. This reduction is more effective than harmonic and linear mistuning patterns proposed in the literature. Also, the optimized mistuning patterns were found to reduce the maximum blade response for all engine excitation orders in the presence of random mistuning. Finally, several conclusions were drawn in regard to minimizing the maximum blade response amplitude by implementing the optimization technique.

II. Bladed Disk Modeling

The lumped-mass–spring damper model used in this study is shown Fig. 1. This model has been used extensively in the literature for studying various aspects of mistuning[10,13,15] and was shown to capture the essential dynamic characteristics of a mistuned bladed disk.[22] The equation of motion for the ith mass representing the ith blade, $i = 1, \ldots, N$, can be expressed as

$$m_i\ddot{x} + \left(c_i^b + 2c_c\right)\dot{x}_i - c_c(\dot{x}_{i-1} + \dot{x}_{i+1}) + \left(k_i^b + 2k_c\right)x_i - k_c(x_{i-1} + x_{i+1}) = f_i(t) \tag{1}$$

where $x_0 \equiv x_N$ is used to describe a closed system. Suppose that only the blade mass is mistuned, and define $m_i = m_b(1 + \delta_i)$, $c_i^b = c_b$, and $k_i^b = k_b$. Assume the coupling damping term $c_c = 0$, and then Eq. (1) can be simplified to

$$(1 + \delta_i)\ddot{x}_i + c_b/m_b\dot{x}_i + \omega_b^2(1 + 2R^2)x_i - \omega_c^2(x_{i-1} + x_{i+1}) = f_i(t)/m_b \tag{2}$$

where $R^2 = \omega_c^2/\omega_b^2$ is defined as the coupling ratio.[10] The engine-order excitation force can be expressed as

$$f_i(t) = F_0 \exp\{[\omega t + (i-1)\theta]j\} \tag{3}$$

The interblade phase angle θ is defined as

$$\theta = 2\pi E/N \tag{4}$$

When Eqs. (2) and (3) are assembled in matrix form, the eigenvalue problem can be solved. (The mode shapes are normalized with respect to the mass matrix.) When proportional damping is assumed, the displacement response amplitude for the ith blade in Eq. (2) can be expressed as

$$x_i(\omega) = \frac{F_0}{m_b}\sum_{k=1}^{N}\frac{\{\phi_k\}^T\mathbf{\Phi}\phi_{i,k}}{\omega_{n,k}^2 - \omega^2 + 2j\xi\omega_{n,k}\omega} \tag{5}$$

where $\mathbf{\Phi}$ is the phase vector defined as $\Phi = (\exp(\omega t), \ldots, \exp\{[\omega t + (i-1)\theta]j\}, \ldots, \exp[\omega t + (N-1)\theta])^T$.

When δ_i is set to 0 in Eq. (2), the displacement $x_i^0(\omega)$ for the tuned case can be obtained by repeating the described process. Denote d as dynamic magnification factor, expressed as

$$d_i(\delta) = \text{maximum}(X_i)/\text{maximum}\left(X_i^0\right), \qquad i = 1, \ldots, N \tag{6}$$

where X_i and X_i^0 are the response amplitudes of the ith blade for the mistuned case and the tuned case, respectively.

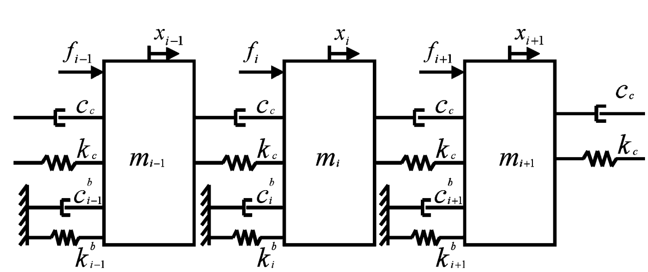

Fig. 1 Lumped, mass-spring model.

III. Optimization Formulation and Response Minimization

A. Problem Formulation

The optimization problem is very complex when the mistuning terms of all of the blades are chosen as independent design variables because the mistuning is a highly nonlinear phenomenon and the blade responses are very sensitive to small changes of blade properties. To minimize the maximum blade response effectively, three issues are considered here:

1) The optimization problem can be formulated to reduce the maximum blade response of an initially mistuned bladed disk system. An intentional mistuning pattern is the only mistuning source in the system. The sensitivity of the maximum blade response to other random mistuning is not included in the optimization but is examined in a Monte Carlo simulation later. This is considered to be an effective way of evaluating the sensitivity to random mistuning.[17,26,27]

2) An initial mistuning pattern may be described as a polynomial function of blade number so that various mistuning patterns can be included. The mistuning pattern is updated iteratively during optimization process. The coefficients of the polynomial are selected as design variables, and thus, the number of variables is equal to the polynomial order. The number of design variables can be reduced dramatically when a lower-order polynomial is used.

3) The determination of an optimized mistuning pattern, suitable for several engine orders, may not be practical because the magnitude of the blade response may depend on the engine excitation orders.[17,28] The optimization problem has to be solved separately for each engine excitation order to achieve the maximum reduction of the blade response.

Based on the response formulation [Eqs. (2–6)], the minimization of the maximum blade response amplitude can be formulated as the following nonlinear constrained optimization problem: Minimize

$$f(\delta) = \text{maximum}(d_i), \qquad i = 1, \ldots, N \tag{7}$$

subject to

$$|\delta_i| \le \delta_i^{\text{limit}}, \qquad i = 1, \ldots, N \tag{8}$$

The mass mistuning term δ_i can be expanded as a polynomial function of the blade number divided by the total number of blades, defined as

$$\delta_i = \sum_{n=1}^{l} a_n \left(\frac{i}{N}\right)^{n-1}, \qquad i = 1, \ldots, N \tag{9}$$

The design variables are defined by the polynomial coefficients a_n. Note that no direct constraints are required for the design variables as long as the relation in Eq. (8) is satisfied. This expansion allows various mistuning patterns to be described in the form of a polynomial with a smaller number of variables. The optimization problem described in Eqs. (7–9) can be solved numerically using the sequential quadratic programming (SQP) method.[29] The detailed formulation and description of the SQP method is beyond the scope of this paper and may be found in the literature.[30–33]

B. Blade Response Minimization

A 12-bladed disk model was used to demonstrate the technique for reducing the maximum blade response by design optimization. The parameters of the nominal tuned system are $k_b = 1$, $k_c = 0.1$, $m_0 = 1$, $\xi = 0.001$, and $F_0 = 1$. The nominal system is a weakly coupled system with $R = 0.01$. The bounds for the blade mass mistuning, δ_i^{limit}, are chosen to be 0.10. The order l of the polynomial was chosen to be 4 in all analyses. When begun with a random initial point superimposed on the tuned system, a series of optimization runs was executed, and the results were evaluated to ensure that an optimized mistuning pattern was obtained for each engine excitation order.

Effect of Engine Excitation Order

Figure 2 shows the envelope of the amplification factor for all of the blades from the optimized mistuning patterns and their

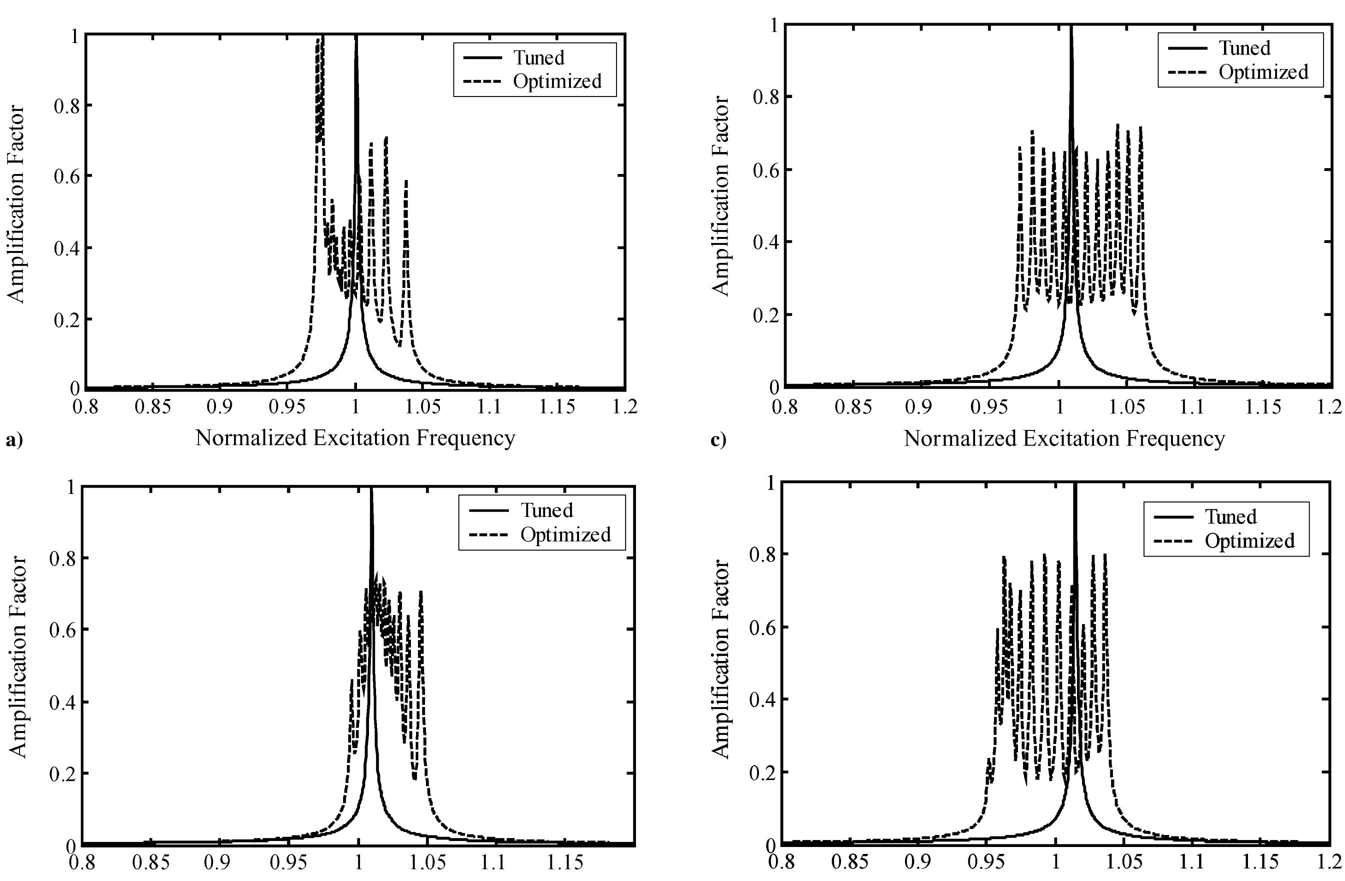

Fig. 2 Amplification factor comparison between the optimized systems with the tuned system: a) $E = 1$, b) $E = 2$, c) $E = 3$, and d) $E = 4$.

comparison with a tuned system under the engine excitation orders 1–4. The excitation frequency was normalized using the blade frequency of the tuned system ω_b. Observe that the dynamic amplification factor for the optimized system can be significantly less than for the tuned system. This reduction in the amplification factor is between 20 and 30% for $E = 2$, 3, and 4. All of the vibration modes of the optimized systems are excited, but the dynamic response levels are less than those in the tuned system. The amplification factor for $E = 1$ after optimization is close to 1, as in the tuned system.

To evaluate the variability of the maximum response reduction with other engine excitation orders, optimization analyses were also carried out for $E = 5$–12. The optimized mistuning patterns obtained for $E = 1$–12 are given in Fig. 3. The optimized mistuning patterns for most engine orders were found to be either an ascending or descending nonlinear curve. The mistuning patterns with harmonic components are for $E = 4$, 5, and 12, but they are in a general descending trend. Interestingly, the optimized mistuning patterns for $E = 3$ and 9 are nearly linear, and this confirms the finding (for $E = 3$ only) from the work by Jones and Cross.[28] Therefore, both the harmonic and linear patterns can be considered as the special cases in the family of optimized mistuning patterns.

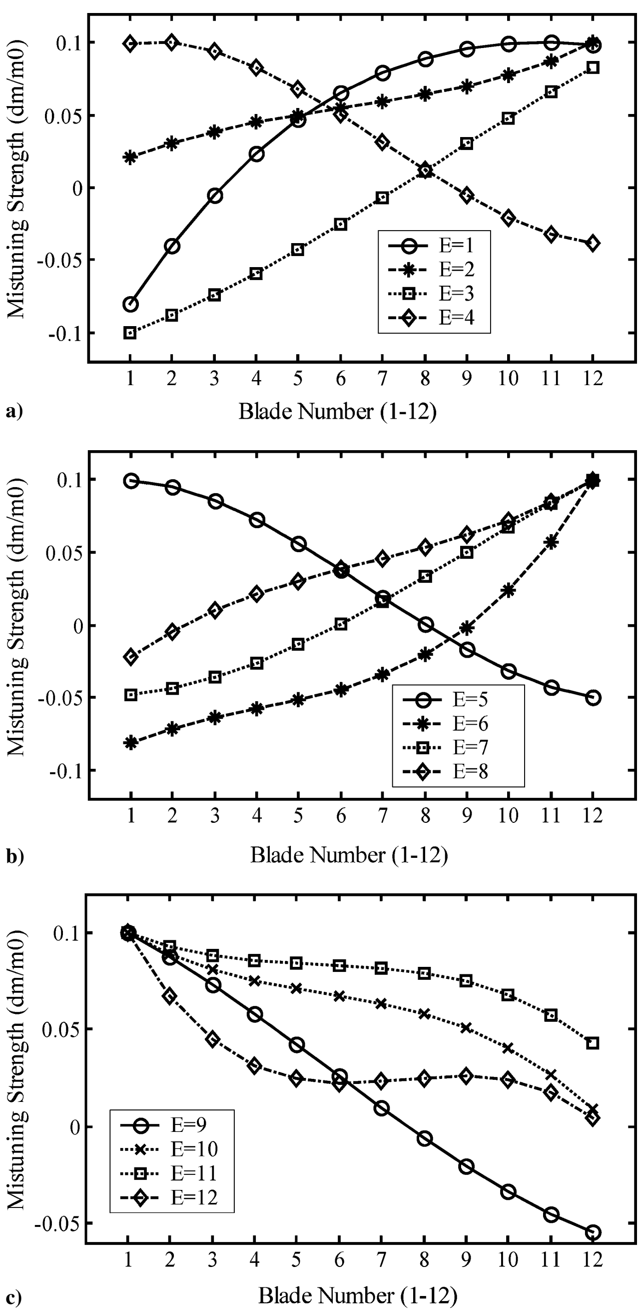

Fig. 3 Optimized mistuning patterns under different engine orders: a) $E = 1$–4, b) $E = 5$–8, and c) $E = 9$–12.

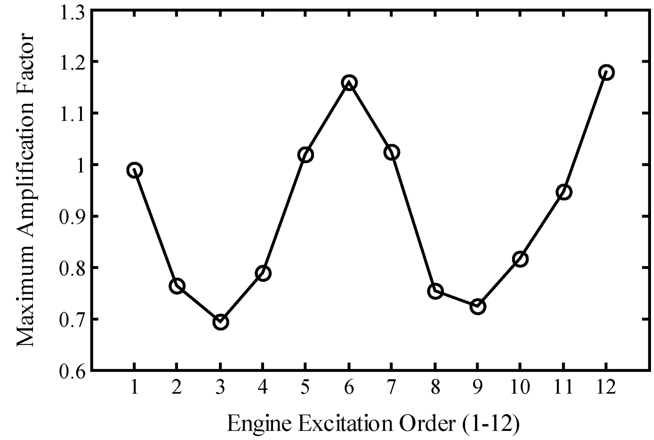

Fig. 4 Relation between amplification factors of optimized mistuning patterns and engine orders.

Figure 4 shows a more complete relation between the amplification factor of the maximum responding blades of the optimized systems and the engine excitation order. The amplification factor of the maximum responding blades exhibits a nearly harmonic pattern with engine excitation order. In comparison with the maximum response in the tuned case, the optimized patterns are most effective for $E = 3$ and 9 (30% reduction for both). However, the optimized mistuning patterns failed to reduce the amplification factor below 1 for $E = 1$, $N/2$, and N. This is consistent with the results reported from a statistical analysis in the literature.[28] When it is noted that the baseline of the system is mistuned through the optimization, it should be appreciated that the optimized mistuning pattern is able to provide a much lower amplification factor than the one from a random mistuning pattern.

Effect of Blade and Disk Coupling

The coupling ratio R is recognized as a key factor in investigating mistuning phenomena because it significantly affects the spacing of eigenvalues and the mode localization due to mistuning.[9] Although the maximum responding blade may change with the coupling ratio,[10] changes in the maximum amplification factor of the blades can be evaluated. The variability of the response reduction with changes in R ratio was examined for 10 different R ratios ($R = 0.05$–0.5 with an interval of 0.05) using optimization analyses for $E = 3$. The optimized mistuning patterns for all R ratios are given in Fig. 5 and exhibit an ascending or descending trend. This trend is typical for the weak coupling case ($R = 0.05$–0.15). Note that the optimized mistuning pattern for $E = 3$ is close to a linear pattern. For the moderate and strong coupling cases ($R = 0.2$–0.5), the optimized mistuning patterns indeed resemble a half-order harmonic pattern. Hence, the shape of the optimized mistuning patterns may depend not only on the engine excitation order but also on the coupling ratio.

Figure 6 shows the relation between the amplification factor of the maximum responding blades from the optimized systems and the coupling ratio. The maximum amplification factor occurs at the weak coupling case. This is consistent with the findings in literature.[10] The amplification factor decreases as the coupling ratio increases to a moderate value and reaches its minimum value (0.60) for $R = 0.25$, providing a 40% reduction in the maximum amplitude. The amplification factor increases dramatically with a further increase in the coupling ratio because the effectiveness of the optimized patterns is compensated by the significant decreasing of the blade response amplitude in the tuned system. Hence, it is highly desirable to design a bladed disk with a moderate disk to blade coupling ratio so that the maximum reduction in amplification factor can be achieved.

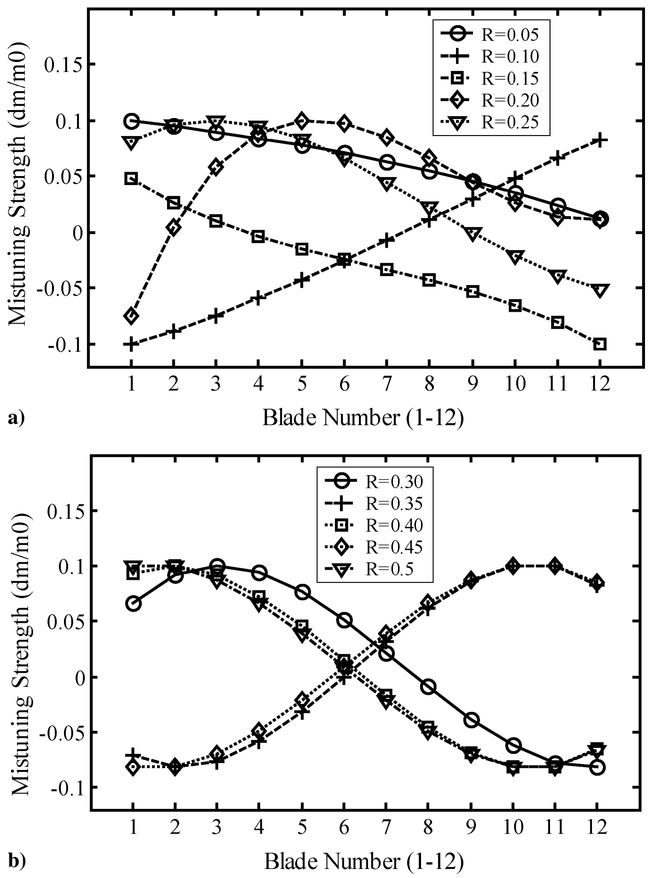

Fig. 5 Optimized mistuning pattern for different *R* ratios: a) *R* ratio from 0.05 to 0.25 and b) *R* ratio from 0.30 to 0.50.

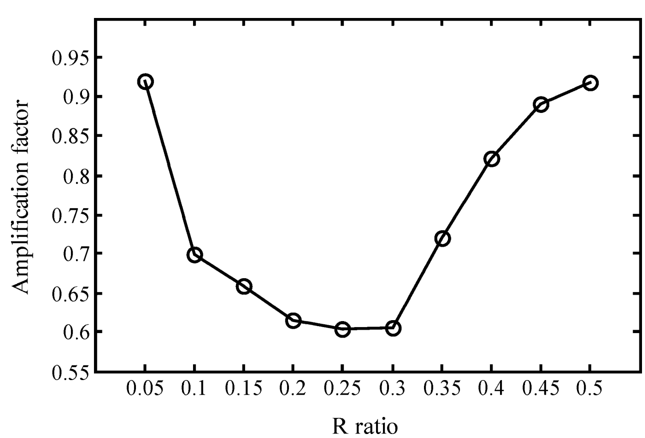

Fig. 6 Relation between amplification factors of optimized mistuning patterns and *R* ratio.

IV. Comparison with Harmonic and Linear Mistuning

To evaluate the effectiveness of the optimized mistuning patterns, the maximum blade response of the optimized system was compared with those from a harmonic mistuning pattern and the linear mistuning pattern proposed in the literature.[17,21,28] The harmonic mistuning pattern was defined[17] as

$$\delta_i = A\cos[2\pi h(i-1)/N], \qquad i = 1, \ldots, N \tag{10}$$

The linear mistuning pattern can be described mathematically as

$$\delta_i = A\{[2(i-1)/(N-1)] - 1\}, \qquad i = 1, \ldots, N \tag{11}$$

For a fair comparison, the amount of intentional mistuning for both the harmonic and the linear mistuning patterns was determined at its most effective value. Figures 7a and 7b show the relation between the amplification factor and the mistuning amplitude for the harmonic and the linear mistuning patterns, respectively ($E = 2$, 3, and 4). For the harmonic mistuning pattern, the six harmonic orders [defined by $h = 1$–6 in Eq. (11)] were examined, and only the most effective harmonic mistuning orders are plotted in Fig. 7a. It is shown that the mistuning strength has a significant effect on the amplitude reduction for both the harmonic and the linear mistuning patterns. In the case of the harmonic mistuning, the harmonic order also affects the amplitude reduction. Figure 8 shows a comparison of the amplification factors of the optimized mistuning pattern with the most effective harmonic and linear mistuning patterns for $E = 3$. As shown in Fig. 8, all 12 vibration modes were excited in the case of the optimized and linear mistuning patterns. For the harmonic mistuning patterns, only six modes were excited for $E = 3$. The smallest values of the amplification factors for the harmonic, linear, and optimized mistuning patterns are given in Table 1.

Table 1 Comparison of amplification factor

Engine excitation order	Harmonic	Linear	Optimized
2	0.966 ($h = 1$)	0.842	0.763
3	0.816 ($h = 2$)	0.745	0.703
4	0.931 ($h = 4$)	0.912	0.790

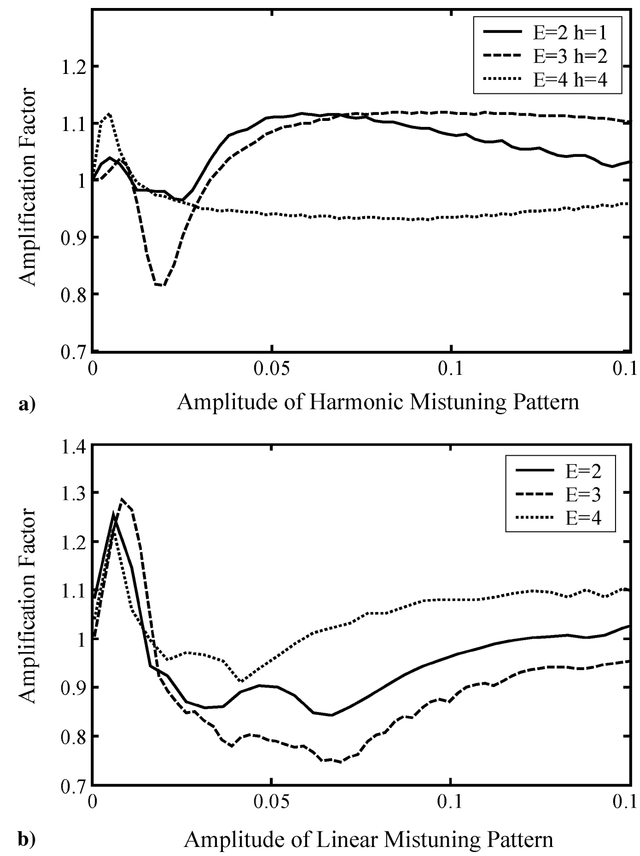

Fig. 7 Amplification factor vs mistuning amplitude: a) harmonic mistuning pattern and b) linear mistuning pattern.

The maximum reduction in the amplification factor is achieved for $E = 3$, by the optimized mistuning patterns (30% reduction). The reduction is 18 and 25% for the harmonic mistuning pattern and the linear mistuning pattern, respectively. Minimum reduction in the amplification factor occurs for $E = 4$ by using the optimized mistuning pattern. The optimized mistuning pattern provides a 21% reduction, which is much more effective than both the harmonic and linear mistuning patterns (7 and 9%, respectively). Therefore, the optimized mistuning pattern provides a more effective reduction in the amplification factor than the harmonic and linear mistuning

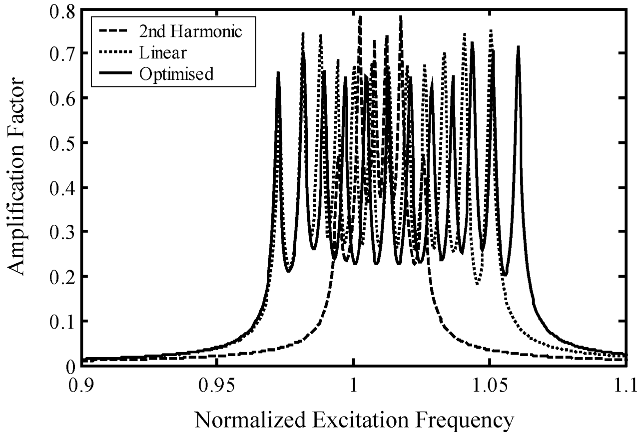

Fig. 8 Comparison of amplification factors, $E = 3$.

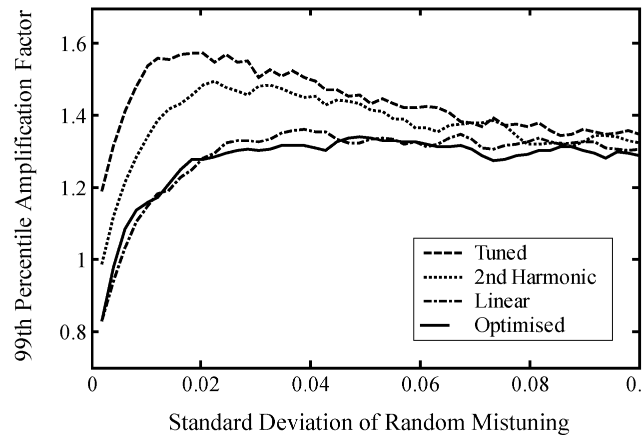

Fig. 9 Comparison of the 99th percentile amplification factor.

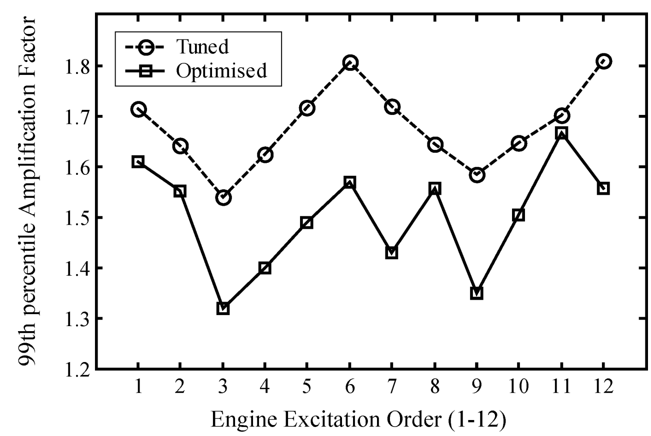

Fig. 10 Comparison of the 99th percentile amplification factor between the tuned system and optimized system.

patterns for each of the engine excitation orders considered. The linear mistuning pattern offers a greater reduction than the harmonic mistuning pattern for $E = 3$, but the advantage is only marginal for $E = 4$. Clearly, both the shape and strength of the mistuning patterns significantly contribute to the effectiveness of the blade amplitude reduction.

Note that the optimized solutions obtained here are satisfactory solutions and that various random starting points were used to avoid reaching local minimums. A fourth-order polynomial was chosen to represent the various mistuning patterns, and this may preclude the further reduction of response amplitude. Nevertheless, the important observation is that the dynamic amplification factor can be reduced effectively by implementing optimized mistuning patterns.

V. Effect of Random Mistuning on the Optimized System

In practice, the existence of random mistuning in a bladed disk is unavoidable due to small differences in blade geometry and material properties caused in the manufacturing process. Intentional mistuning must be able to reduce the maximum blade response in the presence of random mistuning for the technique to be useful. The effect of random mistuning on the dynamic amplification factor of the optimized system was examined by performing a series of Monte Carlo simulations. Mistuning patterns were generated randomly by selecting a blade mass variation with a normal distribution having a mean value of zero and a standard deviation from 0.002 to 0.1. The random mistuning patterns were then superimposed on the optimized mistuning pattern. For each standard deviation, 200 random mistuning patterns were generated, and the models were analyzed to determine the maximum blade response. A type 3 Weibull distribution was fitted to the set of amplification factors in line with the early work in the literature (see Ref. 22). The 99th percentile values of the amplification factors were calculated from the Weibull distribution and is shown in Fig. 9 vs the standard deviation of the random mistuning for $E = 3$. For comparison purposes, the 99th percentile values of the maximum blade response from both the harmonic mistuning pattern and the linear mistuning pattern are also shown in Fig. 9.

The dynamic amplification factor of the optimized system, in the presence of random mistuning, is significantly less than that of the tuned system. This reduction is most effective for a standard deviation of random mistuning in the range from 0 to 0.06 and is less effective when the standard deviation is greater than 0.06. The optimized mistuning pattern reduces the amplification factor more effectively than the second-order harmonic mistuning pattern and provides a greater reduction in the range of the standard deviation of the random mistuning from 0.002 to 0.05. The peak amplification factor is reduced to 1.32 compared with 1.5 for the harmonic mistuning pattern. Compared with the linear mistuning pattern, the optimized mistuning pattern provides a very similar reduction for all of the standard deviations of the random mistuning pattern except that of the peak amplification factor is slightly less (1.32 for the optimized mistuning pattern and 1.35 for the linear mistuning pattern, respectively). This confirms a postulated statement in the literature[28] that a linear mistuning pattern may be close to optimal for $E = 3$.

The response sensitivity to random mistuning for other engine orders was also examined. Figure 10 shows the comparison of the 99th percentile values of the maximum amplification factor for the tuned system and the optimized system for 12 engine excitation orders. Surprisingly, the optimized mistuning patterns are found to be able to reduce the response sensitivity to random mistuning for all of the 12 engine excitation orders, although the minimized values of the amplification factor from these optimized mistuning patterns, in absence of random mistuning, are similar to or higher than those in the tuned system (for $E = 1$, $N/2$, or N). Compared with the tuned system, the optimized mistuning pattern provides the lowest peak response for $E = 3$ and 9 (1.32 and 1.33, respectively) and also provides a significantly lower peak amplification factor for $E = 6$ and 12 (1.56 and 1.58, respectively). Based on the formula of maximum blade response amplitude,[4,15] the maximum amplification factors for a 12-bladed blisk can be as high as 2.23. It is evident that the implementation of an optimized mistuning pattern would be useful for reducing the blade vibration levels and, therefore, extending blade HCF life.

VI. Conclusions

In this paper, the reduction of the maximum blade response was investigated through design optimization. The problem was formulated as a constrained nonlinear optimization process. When a polynomial function was used to describe the mass mistuning, an intentional mistuning pattern to produce the smallest maximum blade response was sought. From this study the following conclusions can be drawn:

1) Describing an intentional mistuning pattern by a polynomial function can reduce the number of design variables dramatically so that the computational costs are only a fractional of those using full-scale optimization of all blade properties.

2) Compared with the tuned system, the optimized intentional mistuning pattern, in the case of a weakly coupled system ($R = 0.01$), is able to reduce the maximum blade response by 20 and 30% for several engine excitation orders. However, the amplification factor for engine orders close to 1, $N/2$, and N cannot be reduced below 1. For a moderately coupled system ($R = 0.25$), the amplification factor can be reduced by as much as 40%.

3) The optimized mistuning pattern exhibits a nonlinear descending or ascending trend. The optimized mistuning pattern is more effective in reducing the amplification factor than the harmonic or linear mistuning patterns proposed in the literature.

4) In the presence of random mistuning, the optimized mistuning patterns were found to be able to reduce the maximum amplification factor for the engine excitation orders that were investigated in the paper. This indicates that the optimized mistuning patterns are effective at reducing the response sensitivity to additional random mistuning within the given range.

Note that this paper does not search for a generic lower bound of amplification factor because the minimal value may vary from one system to another due to the complex nature of the problem. An ideal mistuning pattern that provides the smallest response amplitude may depend on the shape of mistuning pattern, mistuning strength, engine excitation orders, and more important the coupling stiffness between the blades and the disk. For further study it is recommended that the procedure developed in this paper be applied to more realistic finite element models of industrial bladed disks.

Acknowledgments

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